# Design of Blade Contour for High Speed Expansion Turbine

A

Project Report
Submitted for the Partial Fulfilment
of the Degree of

**Bachelors of Technology**
in
**Mechanical Engineering**

*By*
**Chandan Kumar Sahu**
**Roll No. 110ME0326**

*Under the Guidance of*
**Prof S. K. Behera**
**Department of Mechanical Engineering**

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

## NATIONAL INSTITUTE OF TECHNOLOGY

## ROURKELA

# CERTIFICATE

This is to certify that this project report entitled **'Design of Blade Contour for High Speed Expansion Turbine'** submitted by **Chandan Kumar Sahu** bearing the Roll No. **110ME0326** for the partial fulfilment of the requirements for the award of the degree of Bachelors of Technology in Mechanical Engineering at National Institute of Technology Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date:

Prof. S. K. Behera
Department of Mechanical Engineering
National Institute of Technology
Rourkela

# ACKNOWLEDGEMENT

I feel myself fortunate for getting an opportunity to work under Prof. S. K. Behera, and for involving me in this product oriented project with a lot of scope. I express my sincere gratitude and indebtedness for his invaluable guidance, encouragement, thoughtful advices and above all for his cooperative attitude. I really enjoyed a lot working on the project and learning Matlab and Solidworks under his guidance.

I am also thankful to my friends Siddharth Meher and Ananda Kumar Behera for assisting me in the project work.

Chandan Kumar Sahu

# ABSTRACT


Present day expansion turbines are becoming more popular as they are meeting growing need for low pressure cycles in various cryogenic process and liquefaction plants. As the performance of a turbine depends on the turbine wheel, this project is aimed at the exhaustive design of the turbine wheel of mixed flow impellers with radial entry and axial discharge. An attempt has been made to design profile of the expansion turbine wheel used in a turboexpander.

To determine the principal dimensions of the turbine wheel, operating speed has been taken from design charts based on similarity principles. And Hasselgruber's technique is used to design the profile of the turbine wheel. The computational process has been validated against a predesigned turbine wheel.

# CONTENTS

# NOMENCLATURE

***A*** : Area of Cross section normal to the direction of flow ($m^2$)

***b*** : Blade Height (m)

***C*** : Absolute velocity of the fluid (m/s)

$C_0$ : Spouting velocity (m/s)

$C_s$ : Velocity of sound (m/s)

***D*** : Diameter (m)

$D_s$ : Specific diameter (Dimensionless)

***h*** : Enthalpy (kJ/Kg)

$k_1$ & $k_2$: Empirical constant to account for difference in discharge & enthalpy between turbine exit and diffuser exit (Dimensionless)

$K_e$ & $K_h$: Free Parameters (Dimensionless)

***M*** : Mach number (Dimensionless)

***m*** : Polytropic index (Dimensionless)

$m_{tr}$ : Mass flow rate (Kg/s)

***N*** : Rotational speed (rps)

$N_s$ : Specific speed (Dimensionless)

***P*** : Power output of the turbine (Watt)

***Q*** : Volumetric flow rate ($m^3$/s)

***r*** : Radius (m)

***r*** : Radial coordinate (m)

***s*** : Entropy (kJ/Kg-K)

***s*** : Meridional Streamlength (m)

***W*** : Velocity of fluid relative to the blade surface (m/s)

$t_{tr}$ : Blade thickness (m)

***t*** : Central Streamlength (m)

***U*** : Peripheral velocity of the rotor (m/s)

***T*** : Temperature (Kelvin)

***w*** : Width (m)

***z*** : Axial coordinate (m)

**<u>Greek Symbols</u>**

$\boldsymbol{\beta}$ : Relative velocity angle (Radian)

$\boldsymbol{\delta}$ : Angle between meridional velocity and axial coordinate (Radian)

$\boldsymbol{\rho}$ : Density ($Kg/m^3$)

$\boldsymbol{\theta}$ : Tangential coordinate (Radian)

$\boldsymbol{\eta}$ : Efficiency (Dimensionless)

$\boldsymbol{\omega}$ : Angular velocity (Rad/s)

**<u>Subscripts:</u>**

***in*** : Nozzle inlet

***0*** : Stagnation condition

***1*** : Nozzle exit

***2*** : Turbine inlet

***3*** : Turbine Exit

***ex*** : Diffuser exit

***th*** : Throat

***D*** : Diffuser

***m*** : Meridional direction

***r*** : Radial direction

***s*** : Isentropic Condition

***hub*** : Hub at turbine exit

***tip*** : Tip at turbine exit

***mean*** : Average of hub and tip

***Pressure***: Pressure contour of blade

***Suction***: Suction contour of blade

# *Chapter 1*

# *Introduction*

The liquefaction of gases is highly essential for storage and transportation purposes and various cryogenic applications. Turboexpanders find application in refrigeration, air conditioning in airplanes, in geothermal heat power cycles and separation of components from natural gas streams. The atmosphere being the only viable resource from which we can harness the gases, it calls for their storage. For storing these gases, liquefaction is necessary which are carried out using expansion turbines after replacing low pressure cycles. Expansion turbines have advantages over others in terms of efficiency, reliability and system integration.

The term turboexpander is used to define an expander and compressor turbomachine as a single unit. A simple turboexpander system consists of the following parts:

1. **Cold End Housing** : The high pressure gas enters into the turboexpander through this.
2. **Nozzle** : The fluid gets accelerated by passing through the nozzle by losing its pressure energy.
3. **Expansion Turbine** : The high velocity fluid leaves the nozzle and impinges on the blades of the properly aligned turbine. The fluid enters radially and leaves axially.
4. **Diffuser** : It is a diverging passage used to convert the kinetic energy of the leaving fluid into pressure energy to facilitate easy gas ejection.
5. **Loading Device** : It can be a generator or a compressor to extract work output of the turbine.
6. **Shaft** : It connects the turbine with the loading device.
7. **Journal Bearings** : It supports the rotor for alignment and takes up load due to imbalance.
8. **Thrust Bearings** : Thrust bearings carry the vertical loads like load due to pressure difference between compressor and turbine ends and rotor weight. The thrust plates along with the shaft collars consists the thrust bearings.
9. **Bearing Housing** : It holds the bearings.
10. **Warm End Housing** : The warm end housing and the cold end housing collectively form the supporting structure.

11. **Seals** : The seals prevent leakage of the fluid from and inside the system.

The turbine is the most important and the most critical part of an expansion turbine. The overall performance of an expansion turbine is decided by the turbine wheel. Besides the performance parameters it also affects the turbulence in the working fluid, vibration of the system and easy manufacturing of the turbine wheel. The whole expansion turbine system may fail if the turbine fails. This crucial role of a turbine calls for a careful yet optimized design of the turbine wheel.

So this project is aimed at designing the turbine wheel completely and developing a model for the turbine so designed. The modelling is essential to confirm if the model generated using the coordinates obtained from the Matlab code is the same as that can be generated by Ghosh [9]. This Matlab code will eliminate the time consuming and cumbersome process of calculating the coordinates of the turbine blade contour. The design procedure of the turbine is divided into two parts: finding the basic dimensions of the turbine and designing the blade profile.

# Chapter 2
# Literature Review

Lord Rayleigh was the first to propose that an expansion turbine can also be used for liquefaction of gases. He suggested the use of an expansion turbine instead of a piston expander for the same where he emphasized that the most important function of the turbine will be refrigeration rather than the power production. The very first works on liquefaction of gases using an expansion turbine were carried out by Davis (1922), at Linde Works in Germany (1934), Kapitza (1939). In the fifties, the liquefaction process started using small gas bearing turboexpanders. Thereon many improvements were incorporated into the turboexpander system by various people and various organizations.

Nowadays the use of turboexpanders became an essential component in the process of liquefaction of gases. And many gases are needed to be liquefied for storage, transportation, separation applications and various low temperature applications.

## 2.1 Turbine Wheel

Akhtar [1] stated that for any turbine, the gas composition, the flow rate, inlet pressure, inlet temperature and outlet pressure are the design parameters which specify the selection. The current section outlines the prior work done in designing the turbine wheel.

During the last two decades, the performance charts gained a lot of importance for presenting the characteristics of turbomachines. The turbine velocity ratio, pressure ratio, flow coefficient factor, the specific speed are a few of them to quote. Balje [3] presented a simple method for calculating the efficiency and other characteristics for radial turbines. Later similarity principles offered a convenient and practical method for analysis of the characteristics.

To design any turbine according to the similarity principles, it is sufficient to control the specific speed and specific diameter. A constant specific speed specifies all the geometrically similar turbines which will develop unit power when working under a unit head. The specific diameter and specific speed completely define dynamic similarity which means that these pairs specify those turbines which will permit similar flow conditions in them.

## 2.2 Specific Speed and Specific Diameter

Specific speed was first used by Balje [3] to classify hydraulic machines. He first used this parameter to design compressors and gas turbines. A set of values for specific diameter and specific speed may be selected in view of maximizing the polytropic efficiency and for obtaining the optimum geometry [1]. A design chart for selection of specific speed and specific diameter has been proposed by Balje [3] which helps in developing the optimal design geometry and for computing the maximum obtainable efficiency corresponding to a pair of values of specific speed and specific diameter.

An $n_s$-$d_s$ diagram [Fig. 3.1] has been reproduced from the work of Balje [3] for radial inflow turbines with a rotor blade angle of 90° for mixed flow type turbines. This chart has been developed for a specific heat ratio of $\gamma$=1.41. The chart need to be modified for a different value of $\gamma$.

The specific diameter and specific speed are often referred as shape parameters or design parameters as they dictate the overall shape of the turbine. Because of the compressibility of the fluids, it is difficult to apply the specific speed criteria. Vavra [17] has shown that the specific speed is independent of the peripheral speed ratio i.e. the ratio of rotor surface velocity and the spouting velocity and the actual turbine dimensions. Hence the specific speed is independent of the Mach number and the Reynolds numbers that occur. So, specific speed does not satisfies the laws of dynamic similarity if the compressibility of the operating fluid is non-ignorable.

## 2.3 Basic Blade parameters

According to Cartwright [6] and Rohlik [15], to avoid excessive shroud curvature the exit tip to rotor inlet diameter is limited to 0.7. Similarly, to avoid excessive hub blade blockage and loss the exit hub to tip diameter should be more than 0.4. According to Cartwright [6] the ratio of the blade height at inlet to the diameter of the wheel lies in the range of 0.02 to 0.6.

At the inlet the peripheral component of the absolute velocity of the turbine depends on the nozzle angle. According to Balje [3], at the exit, the peripheral component of the absolute velocity is a function of the peripheral speed at the outlet and the blade angle at the exit. Whitfield [20] has shown that, the absolute flow angle can be controlled to minimize the absolute Mach number irrespective of the incident angle. Generally the angle of incidence is a function of number of blades. It lies in the range of -20° to -30°. Hence the absolute flow angle can be controlled to minimize the Mach number at inlet, which is usually selected to lie between 70°-80°.

## 2.4 Number of Blades

Balje [3] has derived an equation to calculate the minimum number of blades in terms of the specific speed. Denton [8] suggested 12 number of blades to avoid flow stagnation on the pressure surface. Rohlik [15] suggested a method to calculate the number of blades taking care of the flow separation in the rotor passage. Twelve complete and twelve partial blades were used by Sixsmith [16] in the medium size helium liquefier designed by him. The number of blades should be adjusted such that the blades with the required width and thickness can be manufactured using the available machine tools.

## 2.5 Diffuser

As the actual velocity of discharge from the wheel, which is the inlet velocity for the diffuser is hardly known, it is difficult to design the diffuser perfectly. The diffuser acts as a compressor, by converting the kinetic energy of the fluid at the turbine exit into potential energy in terms of pressure rise. The expansion ratio is thus increased. It also increases the efficiency.

With regular inlet conditions, the efficiency of a conical diffuser is about 90%, which is obtained with a semicone angle of 5°-6°. Shepherd suggested that the optimum semi cone angle lies between 3°-5°. On increasing the semicone angle, the length of the diffuser will decrease thereby decreasing the friction losses but increasing the chances of flow separation. Balje [3], Whitfield and Baines [19] have suggested a chart for pressure recovery factor versus the geometrical parameters of the diffuser. Ino et al [11] suggested a half cone angle of 5°-6° and an aspect ratio of 1.4-3.3 for an effective diffuser design.

According to Came [5], the inner radius of the diffuser is chosen 5% more than the impeller tip radius and the exit radius of the diffuser as about 40% more than the impeller tip radius. It has also been suggested that the radius of the diffuser at its exit should be designed such that the exhaust velocity should be 10-20m/s. Quack [14] suggested that 30-40% of the residual energy which contains 4-5% of the total energy can be recovered by the downstream diffuser.

Kun and Sentz [13] have suggested a way to design the gross dimensions of the diffuser starting from diffuser discharge piping, eye tip geometry and the rest of the space envelope.

## 2.6 Determination of Blade Geometry

A method for designing the blade profile has been developed by Hasselgruber [10], which has been employed by several successors. For designing the turbine rotor blade geometry, a complete aerodynamical analysis of the flow path has been carried out by Bruce [4]. A series of mean line distributions are used to determine a series of three dimensional streamlines which comprises the rotor blade profile. The turbine blade profile consists of a set of radial and axial coordinates which connects the outlet radii to the inlet radii.

Casey [7] used a computational method using Bernstein Bezier polynomial patches to develop the geometrical shape of the flow channel. Baines and Wallace [2] used one dimensional design calculations to determine the outer dimensions of the rotor, the casing and the blade angles. Krain [12] developed a computer aided design (CAD) method for the same by taking care of computational, manufacturing and aerodynamic aspects for radial ending and backswept centrifugal impellers.

## 2.7 Blade Contour

A general rotor design includes pattern of specification for blade and hub geometry, structural and aerodynamic analysis and iteration of the geometry until acceptable structural and aerodynamic criteria are achieved. The geometry generation must focus on both blade shape and hub geometry.

The feasibility of construction of a turboexpander must be taken into care for determining the best geometry for a radial turboexpander. The effect of various dimensional parameters of impeller on the turbine performance was investigated by Watanabe et al [18]. The parameters were optimized by Leyavorski [24] for a radial centripetal turboexpander operating at low temperature. Balje [3] summarized the various parameters affecting the curvature of flow path in peripheral and meridional planes. He also deduced their effect on boundary layer growth and separation of the flow path. For optimizing the blade profile he has followed a pressure balanced flow path.

# Chapter 3
# Determination of Blade Geometry

The design procedure of turbine wheel is divided into two sections: design of the basic dimensions of the expansion turbine and the turbine blade contour design.

## 3.1 Basic Dimensions

The turbine is designed according to the method suggested by Balje [3], Kun and Sentz [13] which are based on similarity principles. The similarity principles state that to design an optimal geometry for the turbine wheel for maximum efficiency for a given Mach number, Reynolds number and specific heat ratio of a given fluid two dimensionless parameters namely specific speed($N_s$) and specific diameter($D_s$) uniquely specifies the major dimensions of the wheel and its velocity triangles at inlet and exit.

Specific Speed $$N_s = \frac{\omega \times \sqrt{Q_3}}{(\Delta \mathrm{h}_{in-3s})^{3/4}} \tag{3.1}$$

Specific Diameter $$D_s = \frac{D_2 \times (\Delta \mathrm{h}_{in-3s})^{1/4}}{\sqrt{Q_3}} \tag{3.2}$$

A pair of values for the specific speed and specific diameter is been selected from figure 3.1 corresponding to an efficiency.

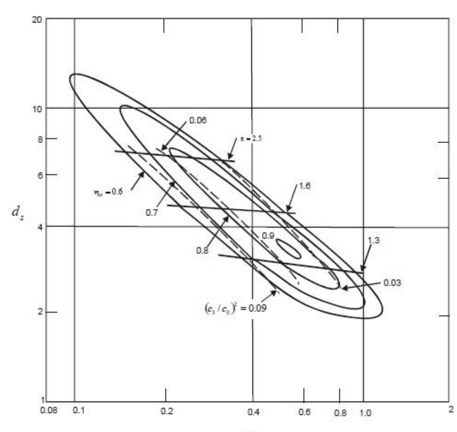
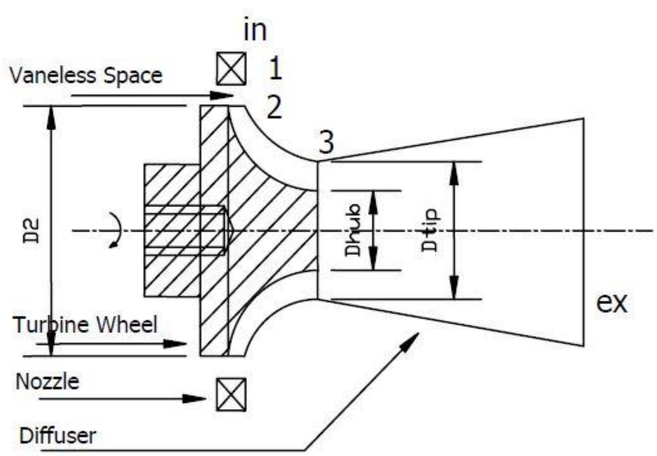



***Figure 3.1:*** *Ns-Ds Diagram for Radial inflow turbines with relative velocity angle at the exit β2=90° (Reproduced from Balje [3] fig. 5.110)*

The chart depicts a set of values for specific speed and specific diameter for various values of efficiency. It can be clearly observed that as the efficiency increases the domain of specific speed and specific diameter gets shortened. A point is chosen on the corresponding curve of required static efficiency and the coordinates of the points are read to get the values of specific speed and specific diameter. The following chart was developed by Balje [3] for a specific heat ratio of $\gamma$=1.41. The chart needs to be modified for any other value of $\gamma$.

The longitudinal cross-section of a turbine is shown in the figure below showing the various state points:




| State Points | |
|---|---|
| in | Nozzle Inlet |
| 1 | Nozzle Exit |
| 2 | Turbine Inlet |
| 3 | Turbine Exit |
| ex | Diffuser Exit |

***Figure 3.2****: Various state points of a turboexpander (Reproduced from Fig 3.2 Ghosh [9])*

In the definition of $N_s$ and $D_s$ the volumetric flow rate $Q_3$ and isentropic enthalpy $h_{3s}$ are at the exit of the turbine which are practically unmeasurable. What is measurable are the volumetric flow rate at the exit of the diffuser $Q_{ex}$ and the enthalpy difference (from thermodynamic tables) between the nozzle inlet (*in*) and the diffuser exit (*ex*) i.e. $\Delta h_{0in-exs}$. Hence to estimate them two empirical factors $k_1$ and $k_2$ are used for volumetric flow rate at turbine exit ($Q_3$) and isentropic enthalpy difference between nozzle inlet (*in*) and turbine exit (*3*) $\Delta h_{0in-3s}$ respectively.

$$\left.\begin{aligned} Q_3 &= k_1 Q_{ex} \\ \rho_3 &= \frac{\rho_{ex}}{k_1} \end{aligned}\right\} \tag{3.3}$$

$$\Delta h_{in-3s} = k_2(\mathrm{h}_{0\mathrm{in}} - \mathrm{h}_{\mathrm{exs}}) \tag{3.4}$$

where $\rho_3$ and $\rho_{ex}$ are the densities at the turbine exit and the diffuser exit.

The factors $k_1$ and $k_2$ account for the difference between the volumetric flow rates or the densities and the enthalpy differences at the turbine exit (*3*) and diffuser exit (*ex*) states. Kun and Sentz [13] has suggested the value of $k_2$ as 1.03. The factor $k_1$ represents the ratio $Q_3/Q_{ex}$ or $\rho_{ex}/\rho_3$, both being equal. The values of $Q_{ex}$ and $\rho_{ex}$ are known. An iterative procedure is followed for the determination of $k_1$.

### 3.1.1 An outline of the iterative procedure to determine $k_1$

An initial guess for the value of $k_1$ has been taken. And using it, the volumetric flow rate and the density of the fluid at the turbine exit can be determined from equations (3.3) and (3.4). Thereafter by substituting the values of $Q_{ex}$, $\Delta h_{in-3s}$ and a pair of values of $N_s$ and $D_s$ in the $N_s$ and $D_s$ equations, we can find out the values of angular velocity of the turbine ($\omega$) and diameter at the turbine inlet ($D_2$). The velocity of the blade tip at inlet ($U_2$) and the spouting velocity ($C_0$) are calculated. Then using the empirical relationships we can determine the hub and tip diameters of the turbine at the exit. Now using the equations (3.11) and (3.12) the mean relative velocity angle ($\beta_{mean}$), mean blade velocity at blade tip ($U_{3mean}$) and the absolute velocity of the blade at the blade exit are determined with the help of blade thickness ($t_{tr}$) and number of blades ($Z_{tr}$).

The diffuser is a part of the turboexpander but not one of the turbine blade. But the design of the diffuser is highly essential to determine the value of $k_1$. The method to determine the basic dimensions of the diffuser is described in the next section. The thermodynamic states of the turbine exit (*3*) are determined from the thermodynamic state of the diffuser exit (*ex*) by using the thermodynamic principles including the density of the fluid at the turbine exit. The error between the empirical value of the density and the determined value of the density is calculated. And a convergence check is carried out. If the error is within limits, then we can accept the value of $k_1$ else we must have to change the value of $k_1$.

On carrying out the iterative procedure we have found the value of $k_1$ as 1.11 with an initial guess of 1. For which the various calculations are shown in the following text.

The input parameters for the turboexpander system are summarized in table 3.2.

| | | | | | |
|---|---|---|---|---|---|
| Working Fluid | : | Air/Nitrogen | Discharge Pressure | : | 1.5 bar |
| Turbine Inlet Temperature | : | 122K | Throughput | : | 67.5 $nm^3/hr$ |
| Turbine Inlet Pressure | : | 6.0 bar | Expected Efficiency | : | 75% |

**Table 3.2 :** *The basic input parameters of the working fluid (Reproduced from Table 3.1 Ghosh [9])*

The actual and ideal thermodynamic parameters at the turbine inlet are summarized in table 3.3.

| | Inlet (State in) | Ideal (Isentropic) exit state (*ex, s*) | Actual Exit State (ex) ($\eta$ =75%) |
|---|---|---|---|
| Pressure (bar) | 6.00 | 1.50 | 1.50 |
| Temperature (K) | 122 | 81.72 | 89.93 |
| Density (Kg/m3) | 17.78 | 6.55 | 5.86 |
| Enthalpy (kJ/Kg) | 119.14 | 80.44 | 90.11 |
| Entropy (kJ/Kg-K) | 5.339 | 5.339 | 5.452 |

**Table 3.3 :** *Parameters found using thermodynamic table (Reproduced from Table 3.2 Ghosh [9])*

All the other necessary parameters for designing the turbine are tabulated in table 3.4.

| **Turbine Design** | | **Diffuser Design** | |
|---|---|---|---|
| Discharge ($m_{tr}$) | 23.26 Kg/s | Maximum permissible diffuser discharge velocity | 20m/s |
| Density at diffuser exit ($\rho_{ex}$) | 5.86 $Kg/m^3$ | Semi cone angle | 5.5° |
| Difference in enthalpies at isentropic and stagnation condtion ($h_{0in}$-$h_{exs}$) | 38.7 kJ/Kg | Inlet diameter of diffuser ($D_{inD}$) | 16.5mm |
| Ratio of tip diameter at turbine exit to turbine inlet diameter ($D_{tip}/D_2$) | 0.676 | Throat diameter ($D_{thD}$) | 11 mm |
| Ratio of hub to tip diameter at turbine exit($D_{hub}/D_{tip}$) | 0.425 | Diffuser exit diameter ($D_{exD}$) | 19.0 mm |
| Number of Blades ($Z_{tr}$) | 10 | Enthalpy at diffuser exit ($h_{ex}$) | 90.11 kJ/Kg |
| Blade thickness ($t_{tr}$) | 0.6 mm | Pressure at diffuser exit ($p_{ex}$) | 1.5 bar |

**Table 3.4**: *Necessary Parameters for Turbine design (Reproduced from Ghosh [9])*

Now, the basic exit parameters for the turbine blade will be

$$\left.\begin{aligned} Q_{ex} &= \frac{m_{tr}}{\rho_{ex}} \\ \rho_3 &= \frac{\rho_{ex}}{k_1} \\ Q_3 &= k_1 Q_{ex} \\ \Delta h_{in-3s} &= k_2(\mathrm{h}_{0\mathrm{in}} - \mathrm{h}_{\mathrm{exs}}) \end{aligned}\right\} \tag{3.5}$$

From Balje's chart, corresponding to an efficiency of 75%, a pair of values for specific speed and specific diameter are selected as

$$N_s = 0.54 \text{ and } D_s = 3.4 \tag{3.6}$$

On substituting these values in the $N_s$ and $D_s$ equations (3.1) and (3.2) respectively, we can find the values of rotational speed and the turbine inlet diameter as

$$\left.\begin{array}{ll} \textit{Rotational Speed} & \omega = 22910 rad/s \\ \textit{Wheel inlet Diameter} & D_2 = 16.0\,mm \\ \text{Tip Speed} & \mathrm{U}_2 = \dfrac{\omega D_2}{2} = 183.28\ m/s \end{array}\right\} \tag{3.7}$$

Now, using the $D_{tip}$ to $D_2$ ratio we can find $D_{tip}$ as 10.8mm and using the $D_{hub}$ to $D_{tip}$ ratio, we can find the value of $D_{hub}$ as 4.6mm. Ghosh [9] has selected the number of blades as 10 and the blade thickness as 0.6mm.

Now from geometrical considerations

$$A_3 = \frac{\pi}{4}(\mathrm{D}_{tip}^2 - D_{hub}^2) - \frac{Z_{tr} t_{tr}(\mathrm{D_{tip}} - \mathrm{D_{hub}})}{2 Sin\beta_{mean}} \tag{3.8}$$

Where $Z_{tr}$=number of blades, $t_{tr}$=Blade Thickness and $\beta$=Exit Blade Angle.

Therefore
$$Q_3 = A_3 C_3 = C_3\left[\frac{\pi}{4}(\mathrm{D}_{tip}^2 - \mathrm{D}_{hub}^2) - \frac{Z_{tr} t_{tr}(\mathrm{D_{tip}} - \mathrm{D_{hub}})}{2 Sin\beta_{mean}}\right] \tag{3.9}$$

$$Q_3 = C_3 \frac{\pi}{4}(\mathrm{D}_{tip}^2 - \mathrm{D}_{hub}^2) - \frac{Z_{tr} t_{tr}(\mathrm{D_{tip}} - \mathrm{D_{hub}})}{2} \times W_3$$

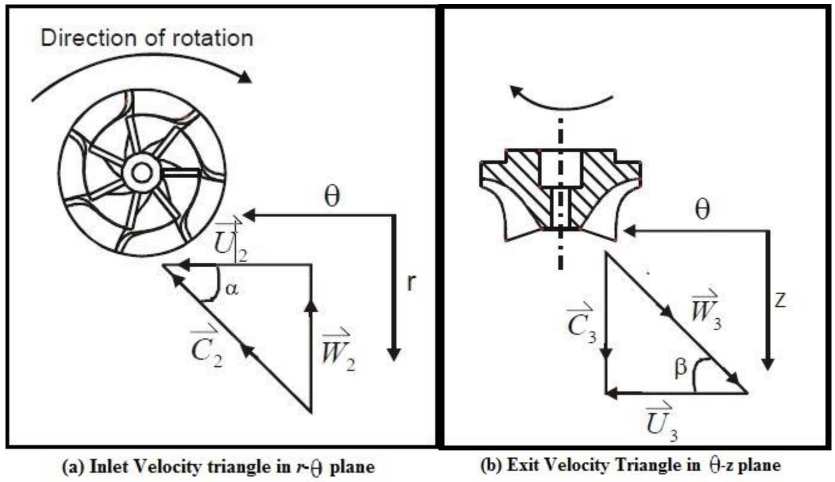



Fig. 3.3: Inlet and Exit Velocity triangles of the turbine wheel (Reproduced from fig.3.3 Ghosh [9])

From the velocity triangle

$$\tan\beta_{mean} = \frac{C_3}{U_{3mean}} = \frac{4C_3}{\omega(\mathrm{D_{tip}} + \mathrm{D_{hub}})} \tag{3.10}$$

On solving the equations (3.11) and (3.12) by substituting the values of Q3, Dtip, Dhub, Ztr, ttr and ω we can find the values of the following

$$\left.\begin{aligned} &C_3 = 90.1 m/s \\ &\beta_{mean} = 45.6° \\ &U_{3mean} = \frac{C3}{\tan\beta_{mean}} = 88.2 m/s \end{aligned}\right\} \tag{3.11}$$

Summary of the basic blade parameters from MATLAB program:

$$\left.\begin{array}{ll} \textit{Rotational Speed} & N = 22910\ rad/s \\ \textit{Wheel Inlet Diameter} & D_2 = 16.0 mm \\ \textit{Eye Tip Diameter} & D_{tip} = 10.8 mm \\ \textit{Eye Hub Diameter} & D_{hub} = 4.6 mm \\ \textit{Number of Blades} & Z_{tr} = 10 \\ \textit{Blade Thickness} & t_{tr} = 0.6 mm \end{array}\right\} \tag{3.12}$$

## 3.1.2 Design of Diffuser

The diffuser is basically an assembly of three sections in series: a converging section or shroud, a short parallel section called the throat and finally a diverging section. The converging section acts like a casing to the turbine. The throat helps in reducing non-uniformity of the flow and the diverging section helps in recovering pressure. The basic dimensions for the diffuser is summarized in table 3.4. The other parameters are

$$\left.\begin{aligned} &\textit{Cross sectional area at the throat} && A_{thD} = \frac{\pi}{4} D_{thD}{}^{2} \\ &\textit{Cross Sectional area at the exhaust} && A_{exD} = \frac{\pi}{4} D_{exD}{}^{2} \\ &\textit{Length of the diverging section} && L_{dD} = \frac{D_{exD} - D_{thD}}{2 \times \tan 5^{\circ}} \end{aligned}\right\} \tag{3.13}$$

**Thermodynamic state at wheel discharge**

The diffuser exit velocity: $$C_{ex} = \frac{Q_{ex}}{A_{ex}} \tag{3.14}$$

Exit stagnation enthalpy $$h_{0ex} = h_{ex} + \frac{C_{ex}^{2}}{2} \tag{3.15}$$

Exit stagnation pressure $$p_{0ex} = p + \frac{1}{2}\rho_{ex} C_{ex}^{2} \tag{3.16}$$

Ignoring the losses in the diffuser, the stagnation enthalpy at the turbine exit $h_3 = h_{0ex}$ *(3.17)*

From the stagnation enthalpy $h_{03}$ and stagnation pressure $p_{0ex}$ , the entropy $s_3$ can be estimated from thermodynamic tables.

Static enthalpy $$h_3 = h_{0ex} - \frac{C_3^{2}}{2} \tag{3.18}$$

From the static enthalpy $h_3$ and $s_3$, the density of the fluid is determined as $\rho_3 = 5.26$ Kg/m$^3$ from the thermodynamic tables.

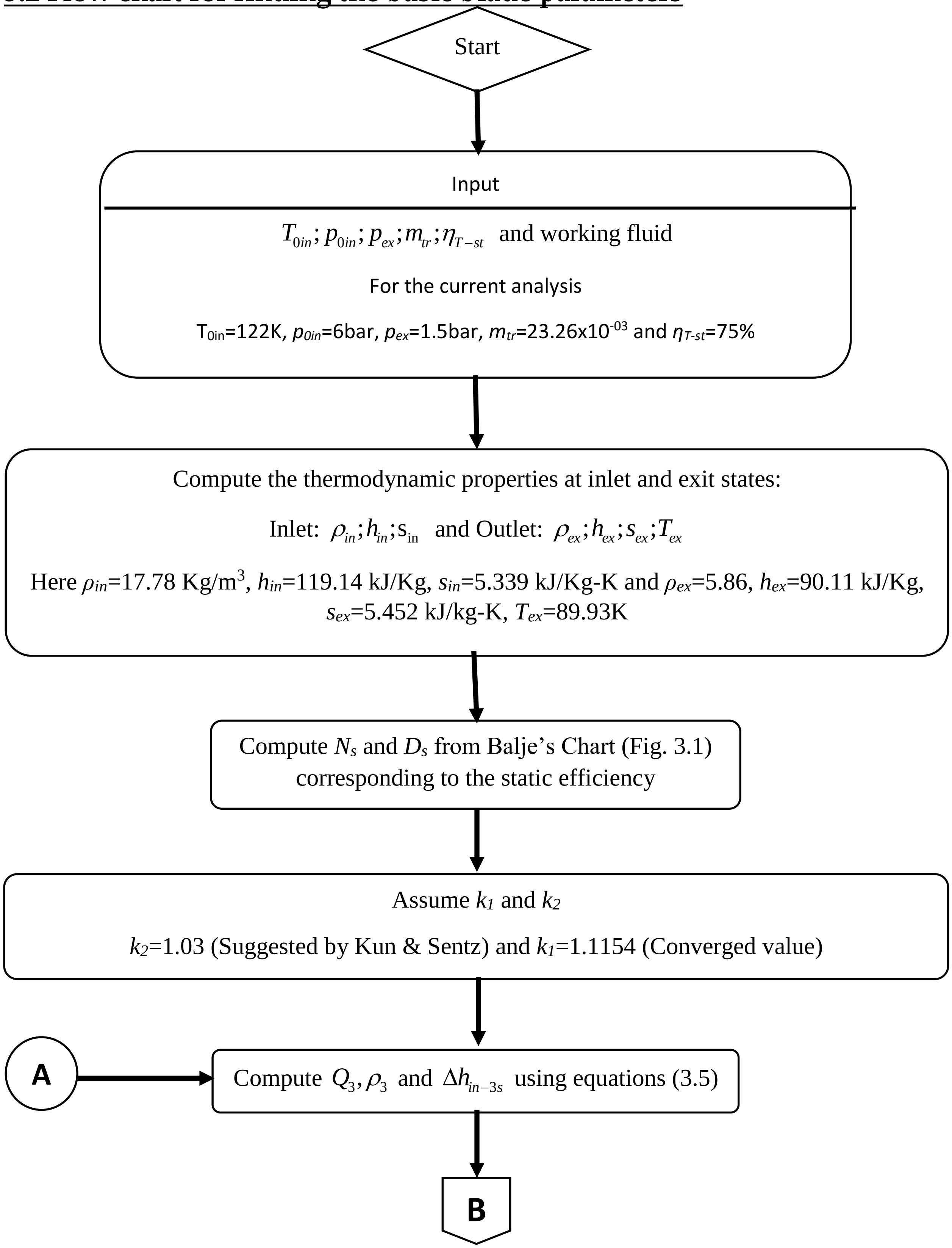
Now, the error is calculated between the so determined density and the density determined empirically. If the error is within the acceptable limits then the value of $k_1$ is accepted.

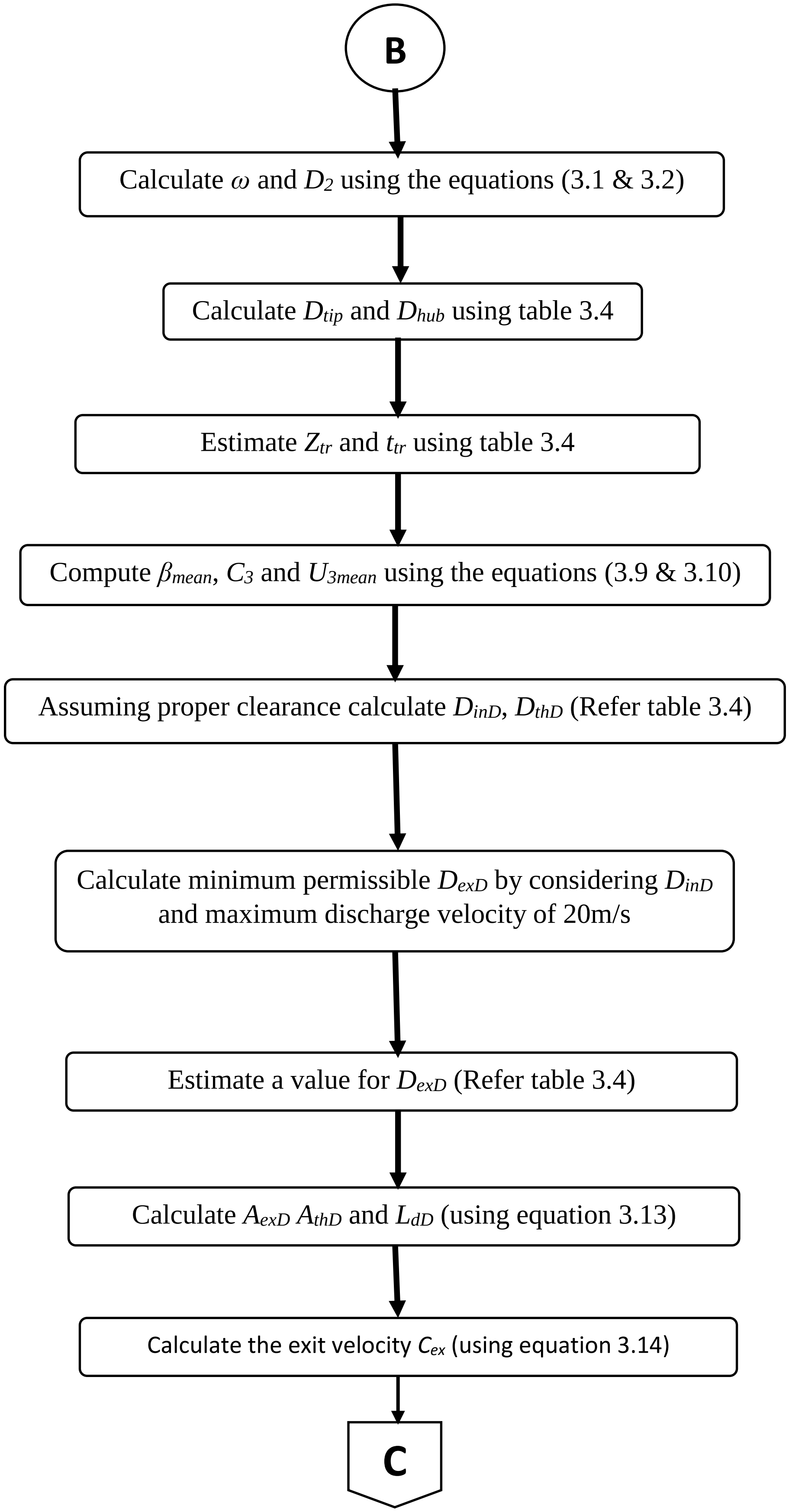
The various thermodynamic properties at the turbine exit (3) are summarized in table 3.4.

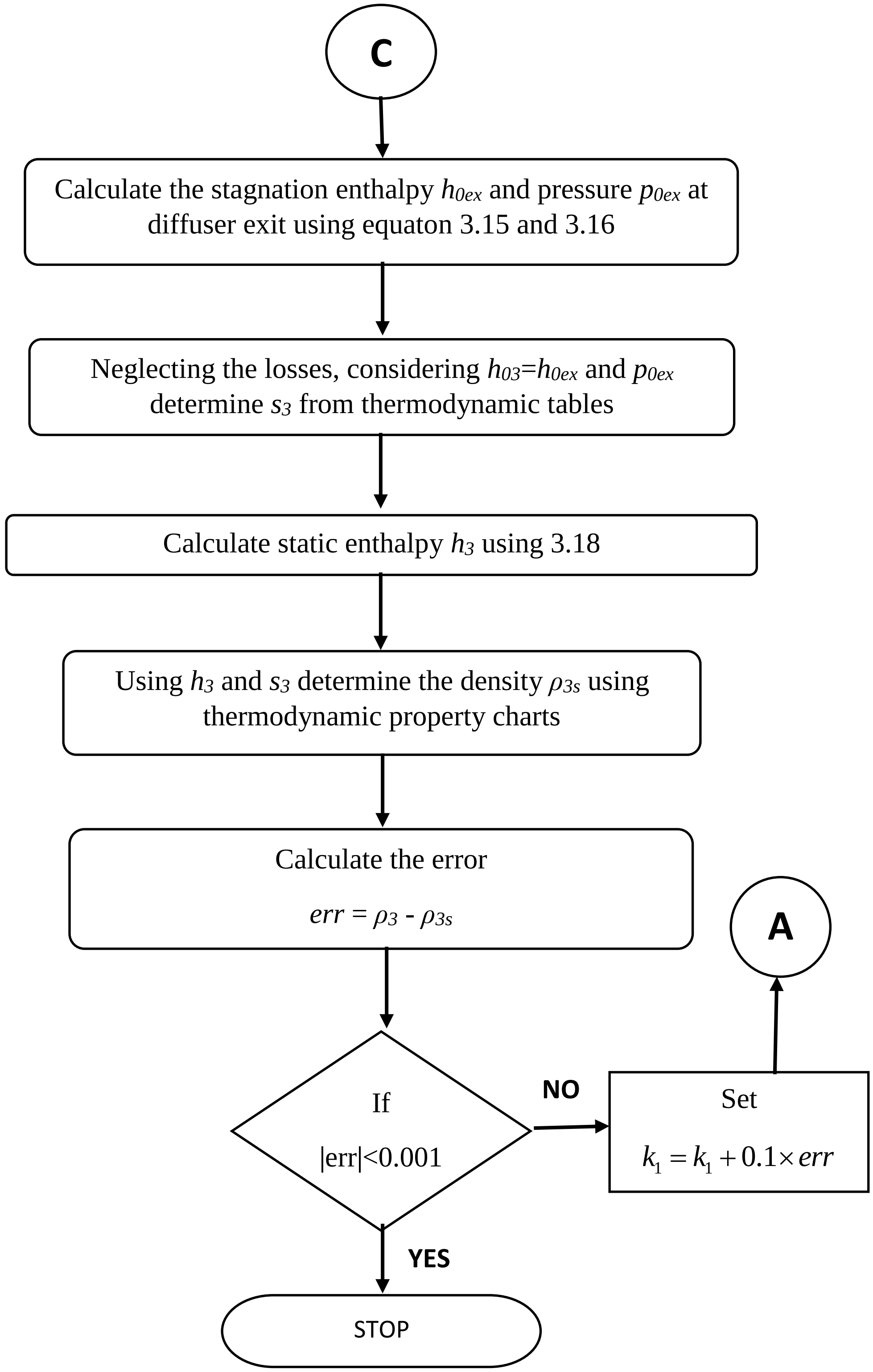
| | Stagnation Value | Static Value |
|---|---|---|
| Velocity (m/s) | 0 | 90.1 |
| Pressure (bar) | 1.505 | 1.29 |
| Enthalpy (kJ/Kg) | 90.2 | 86.15 |
| Entropy (kJ/Kg-K) | 5.452 | 5.452 |
| Temperature (K) | 90.02 | 85.96 |
| Density (Kg/m3) | 5.89 | 5.26 |
| Velocity of Sound (m/s) | 188.87 | 184.4 |
| Viscosity (Pa.s) | 6.13 x 10-6 | 5.90 x 10-6 |

**Table 3.5** : Static and stagnation values at turbine exit (Reproduced fig. 3.3 Ghosh [9])

## 3.2 Flow chart for finding the basic blade parameters




B
Calculate $\omega$ and $D_2$ using the equations (3.1 & 3.2)
Calculate $D_{tip}$ and $D_{hub}$ using table 3.4
Estimate $Z_{tr}$ and $t_{tr}$ using table 3.4
Compute $\beta_{mean}$, $C_3$ and $U_{3mean}$ using the equations (3.9 & 3.10)
Assuming proper clearance calculate $D_{inD}$, $D_{thD}$ (Refer table 3.4)
Calculate minimum permissible $D_{exD}$ by considering $D_{inD}$ and maximum discharge velocity of 20m/s
Estimate a value for $D_{exD}$ (Refer table 3.4)
Calculate $A_{exD}$ $A_{thD}$ and $L_{dD}$ (using equation 3.13)
Calculate the exit velocity $C_{ex}$ (using equation 3.14)
C




***Note:*** *Refer the appendices for the Matlab code based on this flow chart.*

## 3.3 Design of Blade Profile

This chapter describes the design of the blade profile exhaustively. The current method of designing a mixed flow impeller with radial entry and axial discharge is aimed at maximizing the performance. The computational procedure suggested by Hasselgruber [10] and extended by Kun and Sentz [13] was adopted to calculate the coordinates of the blade profile.

The pressure lost by the fluid in the flow passage depends on the length and curvature of the flow path. Thus to account for these losses Hasselgruber [10] defined two flow parameters to control the length and curvature of the flow path. The optimal blade profile defines the magnitude of the velocity and change in its direction. For the turbine blade design the parameters are $K_e$ which varies between 0.75 and 1 and $K_h$ which varies between 1 and 20.

According to Ghosh [9] for selecting the proper combination of the path control parameters $K_e$ and $K_h$, the efficiency, features like uniform and steady operating conditions and easy manufacturing of blades must be taken care of. Upon selection of proper values for these parameters, the tip and hub streamlines(rotor contour) and the change of the flow angle with the flow path coordinate can be determined considering the path to be a pressure balanced flow path. Ghosh [9] also suggested that a random selection of rotor contour and angular change with flow path coordinate may yield a design with potentially high transverse pressure gradients.

This implies that a complete balance of the pressure in the flow path will be practically very difficult to achieve. But by minimizing the relative velocity gradient one can minimize the 3D effects by providing a high number of blades in the region where the pressure and suction streamlines begin to diverge. That's why a set of partial blades are used in some turbomachines. This is to be done till the flow path inclination angle (δ) approaches 90°.

### 3.3.1 The Coordinate System

The blade profile is designed according to Hasselgruber's approach. The equations are derived in terms of an orthogonal coordinate system fitted to the body ($t,b,n$). The coordinates $t$, $b$ and $n$ denotes the direction along the streamline, the lateral coordinate between the pressure and suction surfaces and the depth of the flow path in the turbine passage respectively. Ghosh [9] also describes a meridional coordinate ($s$) to correlate the body fitted coordinate system to a global orthogonal coordinate system ($r, \theta, z$) where the meridional coordinate ($s$) lies on the $r - z$ plane.

## 3.3.2 Assumptions

The blade profile has been designed according to the technique of Hasselgruber [10], which was also employed by Ghosh [9], Balje [3], Kun and Sentz [13],. The assumptions of the Hasselgruber's technique of designing the blade profile are:

(i) Constant acceleration of the relative velocity

The blades of pure radial impellers are so designed that the acceleration of the relative velocity is constant. And this acceleration of the relative velocity ($W$) from wheel inlet to outlet follows a power law relation. Under steady state conditions, the derivative of the relative velocity results

$$\frac{DW}{D\tau} = W\frac{\partial W}{\partial t} = C_1 t^{K_e - 1} \tag{3.19}$$

Where't' refers to the distance along the central streamline in the body fitted coordinate system and '$\tau$' refers to the time coordinate.

On integrating equation (3.20) and substituting the boundary conditions: $W=W_3$ at $t$=0 and $W=W_2$ at $t=t_2$

The solution will be $$W^2 = W_3^2 + (W_2^2 - W_3^2)\left(\frac{t}{t_2}\right)^{K_e} \tag{3.20}$$

(ii) Along the direction normal to the mean relative streamline, there is constant pressure over the blade channel, which indicates that the hydrostatic pressure has negligible effect i.e. $\frac{\partial p}{\partial n} = 0$.

The above two assumptions guide the width of the channel and depth of the curve in the meridian section.

(iii)At the wheel inlet, the relative flow angle $\beta_2 = 90°$

This assumption depicts the shape of the curve in the circumferential direction. Along the flow path, the variation of the relative velocity angle $\beta$ follows the relation

$$\cos ec\beta = \frac{\partial t}{\partial s} = \cos ec\beta_2 + (\cos ec\beta_3 - \cos ec\beta_2)\left(1 - \frac{s}{s_2}\right)^{K_h} \tag{3.21}$$

The meridional velocity at the wheel inlet and exit are equal.

Hence the meridional velocity ratio: $k_I = \dfrac{C_{m3}}{C_{m2}} = 1$ (3.22)

## 3.3.3 Governing equations

The various input parameters like major dimensions of the turbine and the flow conditions at the wheel inlet and outlet are essential for designing the blade profile. To calculate the coordinates $r, \theta, z$ the meridional streamlength $s$ is taken as the independent variable as it correlates the body fitted and the global coordinate systems.

The integration proceeds from the blade exit to the inlet with the following boundary conditions at exit

$$s = s_3 = 0\ ,\ r = r_3\ ,\ z = z_3 = 0\ ,\ \delta = \delta_{outlet} \tag{3.23}$$

till $s = s_2$ . The process terminates at $r = D_2/2$ and $\beta_2 = 90°$ .

The Hasselgruber's formulation leads to three characteristic functions:

$$f_1\left(\frac{s}{s_2}\right) = \sqrt{\left(\mathrm{cosec}(\beta_{\mathrm{mean}})\right)^2 + A \times \left\{\left(\cos ec(\beta_2)\right)^2 - \left(\cos ec(\beta_{\mathrm{mean}})\right)^2\right\}} \tag{3.24}$$

Where $$A = \left[\frac{\frac{s}{s_2} \times (\mathrm{k_h}+1) \times \cos ec(\beta_2) + \left(\cos ec(\beta_{\mathrm{mean}}) - \mathrm{cosec}(\beta_2)\right) \times \left\{1 - \left(1 - \frac{s}{s_2}\right)^{k_h+1}\right\}}{k_h \times \cos ec(\beta_2) + \mathrm{cosec}(\beta_{\mathrm{mean}})}\right]^{k_e} \tag{3.25}$$

And $$f_2\left(\frac{s}{s_2}\right)=\frac{1}{\mathrm{cos}ec(\beta_2)+\left\{\mathrm{cos}ec(\beta_{\mathrm{mean}})-\mathrm{cosec}(\beta_2)\right\}\times\left(1-\frac{s}{s_2}\right)^{k_h}} \tag{3.26}$$

$$f_3\left(\frac{s}{s_2}\right)=f_1\left(\frac{s}{s_2}\right)\times\sqrt{1-f_2^2\left(\frac{s}{s_2}\right)} \tag{3.27}$$

The variation of the relative acceleration of the fluid from the wheel inlet to the exit is given by the function $f_1$. The relative flow angle along the flow path is depicted by the function $f_2$. And the function $f_3$ is a function of $f_1$ and $f_2$.The radius of curvature of the flow path is a function of all the three characteristic functions $f_1$, $f_2$ and $f_3$ given by

$$R_m=\left[\frac{f_1\left(\frac{s}{s_2}\right)\times f_2\left(\frac{s}{s_2}\right)}{\frac{r}{r_{mean}\times\tan(\beta_{\mathrm{mean}})}-f_3\left(\frac{s}{s_2}\right)}\right]^2\times\frac{r}{\cos(\delta)} \tag{3.28}$$

Where $$r_{mean}=\frac{r_{hub}+r_{tip}}{2}=\frac{D_{hub}+D_{tip}}{4} \tag{3.29}$$

The angle between the meridional velocity and the axial coordinate is given by

$$\delta=\int_0^s\left(\frac{1}{R_m}\right)ds \tag{3.30}$$

And the coordinates $r,\theta,z$ of the central streamline are given by

$$\left.\begin{aligned} r&=\int_0^s(\sin\delta)ds\\ \theta&=\int_0^s\left(\frac{\sqrt{1-f_2^2\left(\frac{s}{s_2}\right)}}{r\times f_2\left(\frac{s}{s_2}\right)}\right)ds\\ z&=\int_0^s(\cos\delta)ds \end{aligned}\right\} \tag{3.30}$$

The coordinates so determined are the coordinates of the mean blade profile which needs offsetting to determine the coordinates of the tip and hub profiles of the blade. This middle stream surface or the mean blade profile is created by joining the points on the tip profile to the corresponding points on the hub profile. The coordinates of the tip and hub profiles are calculated by using channel depth and the angle between the meridional component of velocity and the axial coordinate.

A characteristic factor is defined to represent the ratio of meridional velocity to circumferential velocity given by

$$\lambda_m = \frac{C_{m3}}{U_3} = \tan \beta_3 \tag{3.31}$$

On the meridional streamline, the velocities at different points are

$$\left.\begin{aligned}
W_m &= C_{m3} \times f_1\left(\frac{s}{s_2}\right) \\
\beta_m &= \sin^{-1} f_2\left(\frac{s}{s_2}\right) \\
U_m &= \frac{r}{r_{mean}} \times U_3 \\
C_u &= U - W \cos \beta \\
C_m &= C_{m3} \times f_1\left(\frac{s}{s_2}\right) \times f_2\left(\frac{s}{s_2}\right) \\
C &= \sqrt{C_m^2 + C_u^2}
\end{aligned}\right\} \tag{3.32}$$

The density along the fluid flow path will be

$$\rho = \rho_3 \times \left(1 + M_3^2 \times \frac{(m-1)\gamma}{2m} \times \frac{U^2 - W^2 - U_3^2 + W_3^2}{C_{m3}^2}\right)^{\frac{1}{m-1}} \tag{3.33}$$

By using the equations 3.34 and 3.35, we can find the channel width and channel depth at each point from the following equations

$$
\left.\begin{aligned}
w_{tr} &= \frac{2\pi r \sin\beta - z_{tr} t_{tr}}{Z_{tr}} \\
\Delta b &= \frac{m_{tr} \sin\beta}{Z_{tr} w_{tr} \rho C_m}
\end{aligned}\right\} \quad (3.34)
$$

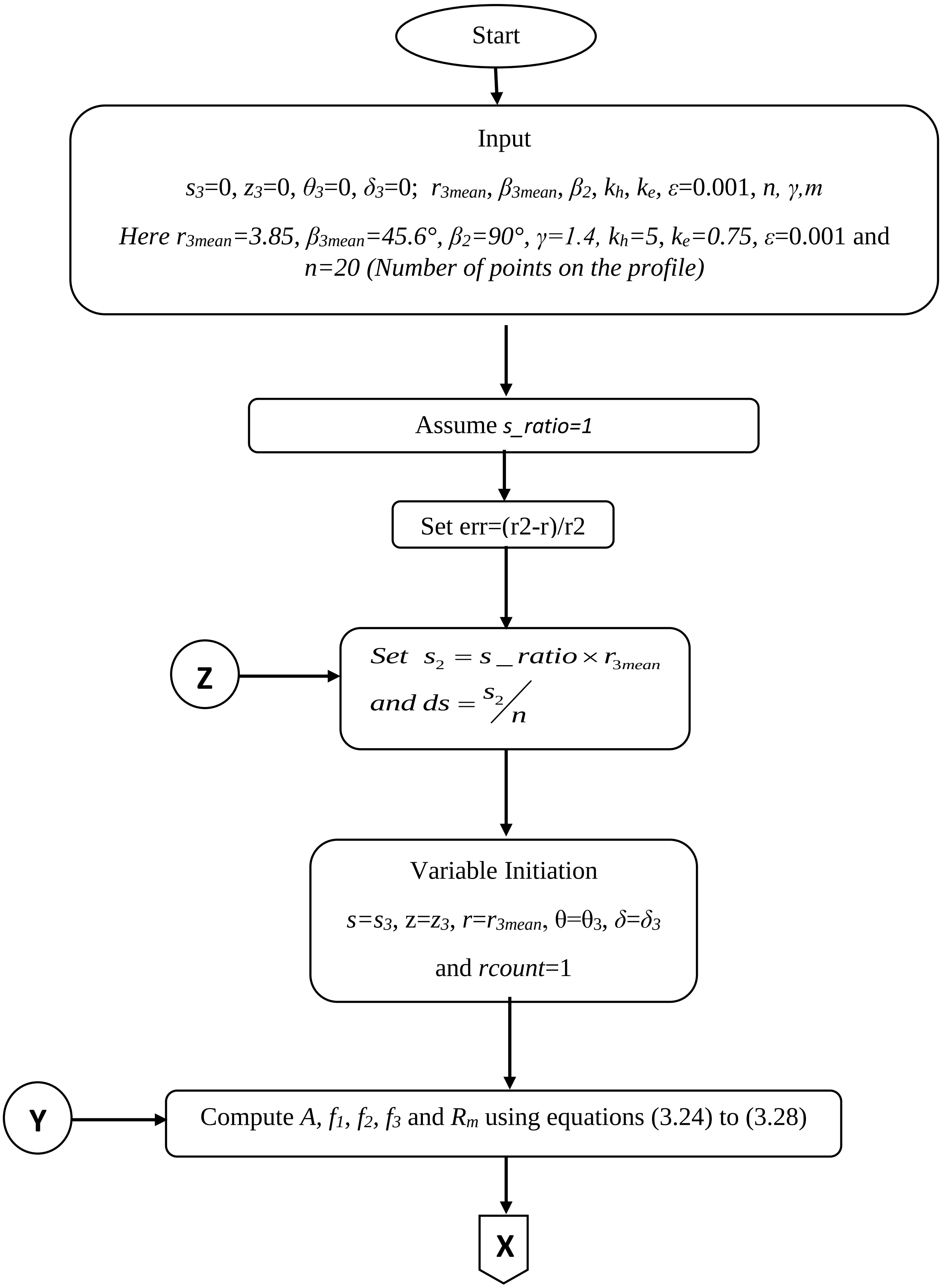
And finally the $r, \theta, z$ coordinates of the tip and hub streamlines can be calculated by using the following equations:

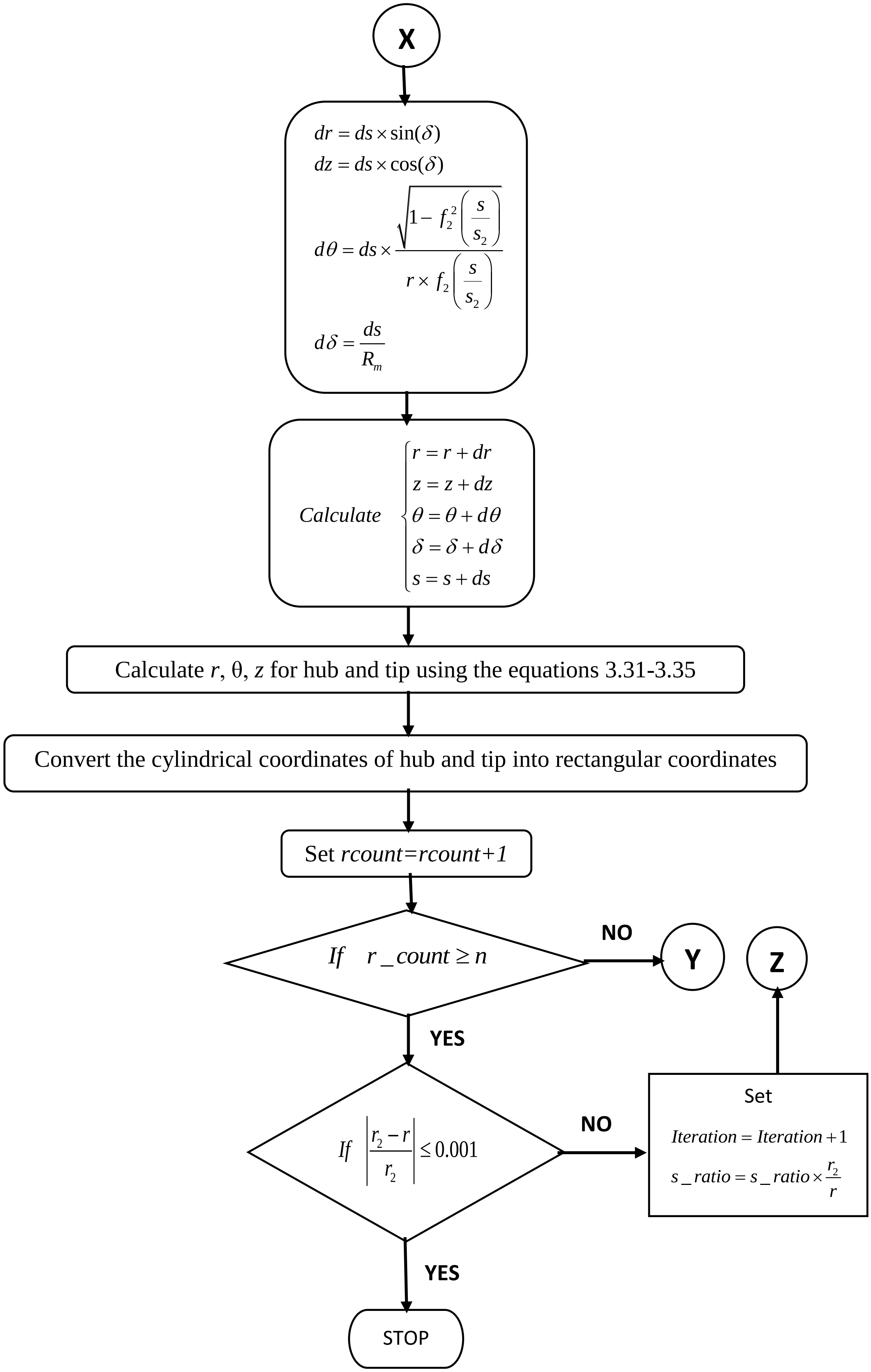
$$
\left.\begin{aligned}
\theta_{tip} &= \theta_{hub} = \theta_{mean} \\
r_{hub} &= r_{mean} - \frac{\Delta b}{2}\cos\delta \\
r_{tip} &= r_{mean} + \frac{\Delta b}{2}\cos\delta \\
z_{hub} &= z_{mean} + \frac{\Delta b}{2}\sin\delta \\
z_{tip} &= z_{mean} - \frac{\Delta b}{2}\sin\delta
\end{aligned}\right\} \quad (3.35)
$$

The surface so generated is the mean surface within a blade whose thickness is negligible. The actual surface is generated by the suction surfaces and the pressure surfaces which are obtained by translating the mean blade surface in +ve and –ve directions by half the blade thickness. The necessary equations are given below

$$
\left.\begin{aligned}
r_{pressure} &= r_{suction} = r_{mean} \\
z_{pressure} &= z_{suction} = z_{mean} \\
\theta_{pressure} &= \theta_{mean} + \frac{t_{tr}}{2 \times r_{mean}}\cos\beta \\
\theta_{suction} &= \theta_{mean} - \frac{t_{tr}}{2 \times r_{mean}}\cos\beta
\end{aligned}\right\} \quad (3.36)
$$

## 3.4 Flow chart for finding the blade contour








***Note:*** *Refer the appendices for the Matlab code based on this flow chart*

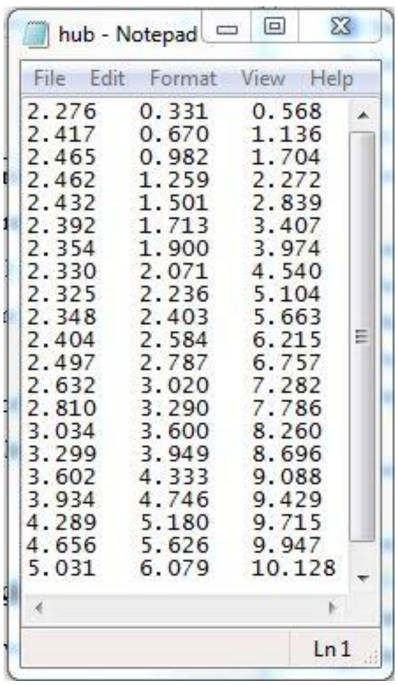
*Chapter 4*

# *Modelling in Solidworks*

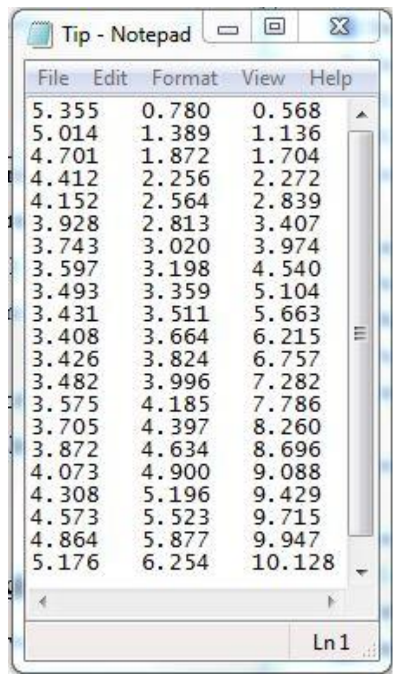
After determining the coordinates of the hub and tip of pressure and suction sides of the blade, it is necessary to create the model of the profile in any CAD software. We have used SolidWorks to generate the 3D model of the turbine. Currently the blade is generated assuming uniform thickness of the blade from inlet to exit.

## 4.1 Ground Work

Generate the coordinates of the hub and tip profiles in rectangular coordinates using Matlab and save them as text files as shown in fig 4.1 and 4.2.




| | | |
|---|---|---|
| 2.276 | 0.331 | 0.568 |
| 2.417 | 0.670 | 1.136 |
| 2.465 | 0.982 | 1.704 |
| 2.462 | 1.259 | 2.272 |
| 2.432 | 1.501 | 2.839 |
| 2.392 | 1.713 | 3.407 |
| 2.354 | 1.900 | 3.974 |
| 2.330 | 2.071 | 4.540 |
| 2.325 | 2.236 | 5.104 |
| 2.348 | 2.403 | 5.663 |
| 2.404 | 2.584 | 6.215 |
| 2.497 | 2.787 | 6.757 |
| 2.632 | 3.020 | 7.282 |
| 2.810 | 3.290 | 7.786 |
| 3.034 | 3.600 | 8.260 |
| 3.299 | 3.949 | 8.696 |
| 3.602 | 4.333 | 9.088 |
| 3.934 | 4.746 | 9.429 |
| 4.289 | 5.180 | 9.715 |
| 4.656 | 5.626 | 9.947 |
| 5.031 | 6.079 | 10.128 |

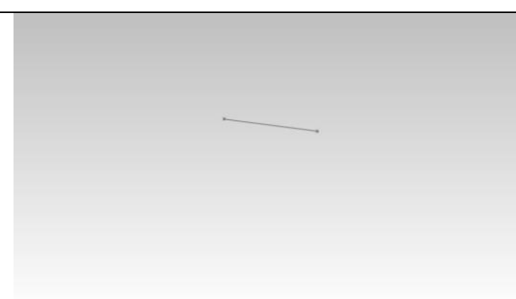


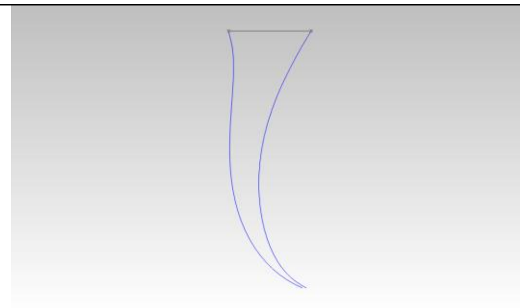
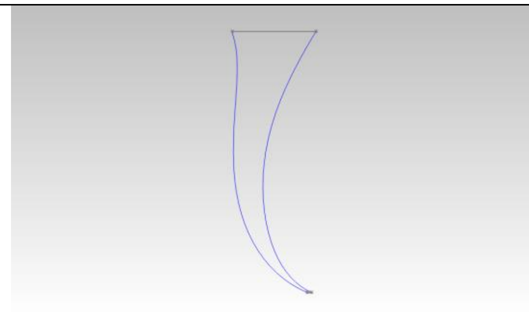
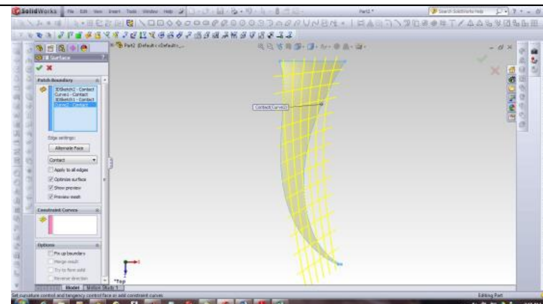
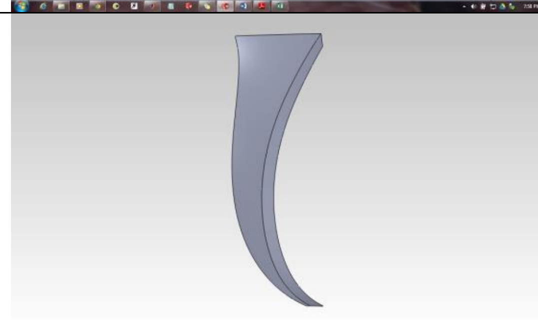
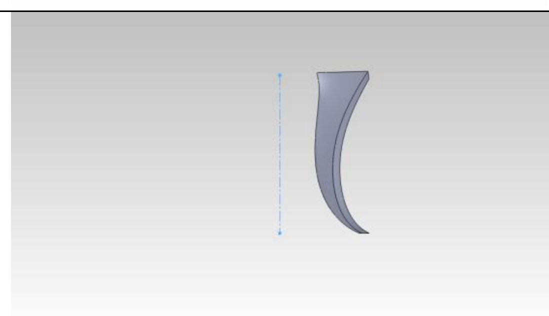

| | | |
|---|---|---|
| 5.355 | 0.780 | 0.568 |
| 5.014 | 1.389 | 1.136 |
| 4.701 | 1.872 | 1.704 |
| 4.412 | 2.256 | 2.272 |
| 4.152 | 2.564 | 2.839 |
| 3.928 | 2.813 | 3.407 |
| 3.743 | 3.020 | 3.974 |
| 3.597 | 3.198 | 4.540 |
| 3.493 | 3.359 | 5.104 |
| 3.431 | 3.511 | 5.663 |
| 3.408 | 3.664 | 6.215 |
| 3.426 | 3.824 | 6.757 |
| 3.482 | 3.996 | 7.282 |
| 3.575 | 4.185 | 7.786 |
| 3.705 | 4.397 | 8.260 |
| 3.872 | 4.634 | 8.696 |
| 4.073 | 4.900 | 9.088 |
| 4.308 | 5.196 | 9.429 |
| 4.573 | 5.523 | 9.715 |
| 4.864 | 5.877 | 9.947 |
| 5.176 | 6.254 | 10.128 |

***Fig 4.1:*** *Coordinates in Text file*

## 4.2 Creating the model in SolidWorks:

The detailed process of developing the model in Solidworks is described in table 4.1 in steps.

| | |
|---|---|
| **Creating the closing line at turbine exit**<br>Create the end points of the closing line at the turbine exit by using the 3D Sketch option and then create the line.<br>Here we have used (2.32, 0.0, 0.24) and (5.38, 0.0, -0.24) to create the line. |  |

| | |
|---|---|
| **Importing the curves to create the blade contour**<br>Import the text files generated earlier to create the blade profile by<br>Insert -> Curve -> Curve through XYZ Points<br>Browse -> Change the file type from *.sldcrv to *.txt and import them |  |
| **Completing the blade profile**<br>Go to 3D Sketch<br>Create two points at the end points of the imported curves. Join them to create a line to close the profile. |  |
| **Creating the mean blade surface**<br>The mean blade surface is created by using the Fill option available in surface modeling features. And it is completed by selecting the surfaces in any order.<br><br>Note: Fill action can't be completed if both the closing lines are created in a single sketch. |  |
| **Creating a single blade**<br>The first single blade is created by using the 'Thicken' option available in solid modelling features by selecting the 'Thicken both sides' option and entering the semi-thickness value in the box |  |
| **Creating the Symmetry axis**<br>Create the symmetry axis by creating an axis using 3D Sketch option then entering the coordinates by making the x and y coordinates of the points at inlet and exit of the hub as zero.<br>Here the coordinates of the inlet and exit of the hub are (2.32, 0, 0.24) and (6.341, 4.745, 7.9). So the coordinates of the axis will be (0, 0, 0.24) and (0, 0, 7.9). |  |

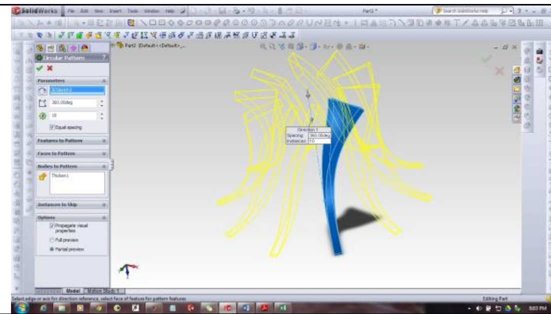
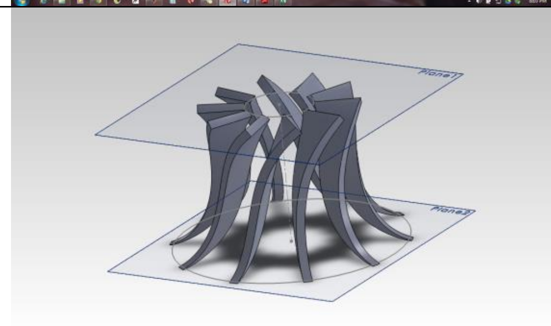
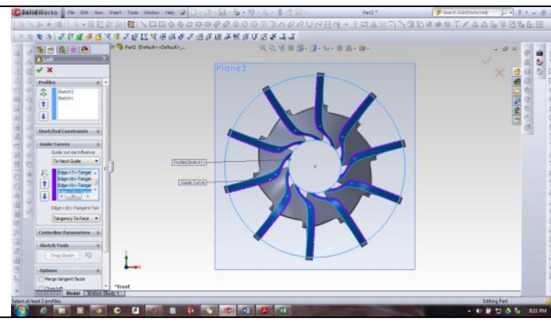
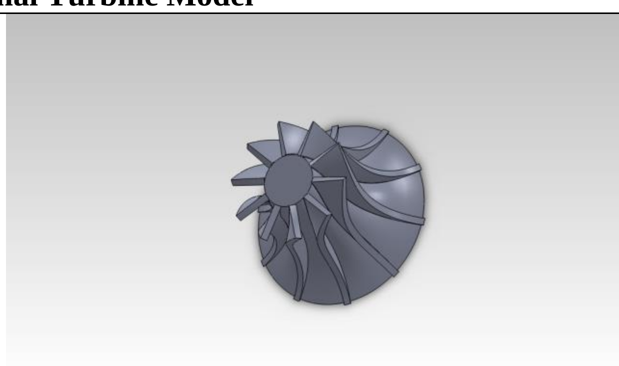
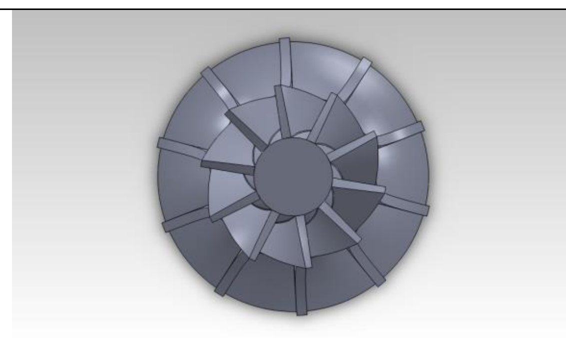
| | |
|---|---|
| **Creating the blades**<br>All the required number of blade are created by using the 'circular pattern' option available in the solid modeling features by specifying the axis, number of instances and the bodies to pattern. |  |
| **Creating the end cross sections**<br>The end cross sections of the blade are created by creating to planes at the ends and then by creating a circle on each plane with hub radii at inlet and exit of the turbine. |  |
| **Creating the hub**<br>The hub of the turbine is created by using the 'Loft' option available in the solid modeling features by selecting the two circular hub cross sections as profiles, selecting one edge from each blade with 'Tangency to curve' option ON and by unticking merge tangent faces, merge results and close loft options. |  |
| **Final Turbine Model** | |
|  |  |

***Table 4.1**: Steps to create the model in Solidworks*

*Chapter 5*

# *Results and Discussions*

The current investigation is aimed at the simplification of the process of design and analysis of the turbine of an expansion turbine used for cryogenic applications. The current analysis was compared with that of Ghosh [9] for a specific efficiency of 75%. However by modifying the code, we can design the turbine for any efficiency.

It is better to write the code in two parts: one for finding the major dimensions of the turbine and another for finding the blade contour. This makes the code convenient to understand and debug.

A model has been created from the current investigation by using Solidworks is shown fig.5.1. The model generated by the Ghosh is shown in Fig. 5.2. There are minor differences in the turbine models which may due to differences in assumption of different parameters.

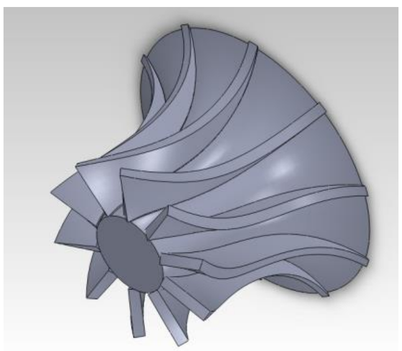


***Fig 5.2****: The model generated from the coordinates obtained from current investigation*

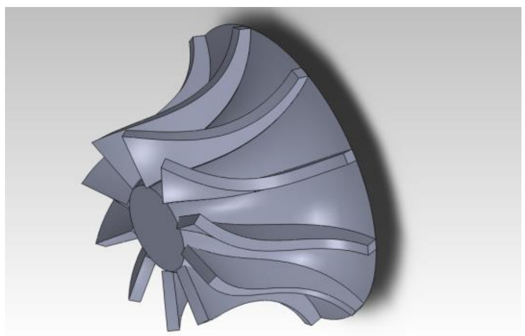


***Fig 5.1****: The model generated from the coordinates generated by Ghosh [9]*

*Chapter 6*

# *Conclusion and Future Work*

The design of the blade contour for high speed expansion turbines can be automated by using a Matlab code. An attempt for which has been made in this project.

This method can be extended for other types of turbines and compressors for different input parameters.

A user interface can also be designed to make the code more user friendly to use. The generation of the model of the turbine in Solidworks is also time consuming. Future work can be carried out for automating the process of modelling in Solidworks using Solidworks API.

# *References*


1. **Akhtar M. S.** Selection and Optimisation of Centrifugal Compressors for oil and gas applications. Using computers in the design and selection of fluid machinery I.Mech.E (1993), 29-41
2. **Baines, N. C. and Wallace, F. J.** Computer aided design for mixed flow turbines for turbochargers Trans ASME J Eng Power (1979) V101, 441-448
3. **Balje, O. E.** Turbomachines John Wiley and Sons (1981)
4. **Bruce, G. J.** Computer-aided Turbomachinery Design System I Mech E Conference on Turbochargers I Mech E Publications (1998)
5. **Came, P. M.** The development, application and experimental evaluation of a design procedure for centrifugal compressors Proc Inst Mech Eng (1978) 192, 49-67
6. **Cartwright, W. G.** Specific speed as a measure of design point efficiency and optimum geometry for a class of compressible flow turbomachines Scaling for performance prediction in rotordynamic machines I Mech E (1978), 139-145
7. **Casey, M. V.** A computational geometry for the blades and internal flow channels of centrifugal compressors Trans ASME J Eng Power (1983) V105, 288-295
8. **Denton, J. D.** The turboexpander - a design, make and test student project ASME-96-GT- 191 (1996)
9. **Ghosh, P.** Analytical and Experimental Studies on Cryogenic Turboexpanders Ph.D dissertation, IIT Kharagpur
10. **Hasselgruber, H.** Stromungsgerechte gestaltung der laufrader von radialkompressoren mit axialem laufradeintrict Konstruction (1958), 10(1) 22(in German)
11. **Ino, N., Machida, A. and Ttsugawa, K.** Development of high expansion ratio Helium turboexpander Advances in Cryogenic Engineering (1992), V37B, 835-844
12. **Krain, H. A.** CAD method for centrifugal compressor impellers Trans ASME J Eng Power (1966) V106, 188-202
13. **Kun, L .C. and Sentz, R. N.** High efficiency expansion turbines in air separation and liquefaction plants International Conference on Production and Purification of Coal Gas & Separation of Air, Beijing, China (1985), 1-21
14. **Quack, H.** Cryogenic expanders Proc ICEC 18, Mumbai (2000)
15. **RohliK, Harold E.** Analytical determination of radial inflow turbine geometry for maximum efficiency NASA TN D-4384 (1968)
16. **Sixsmith, H.** Miniature expansion turbines, C A Bailey (Ed), Advanced Cryogenics Plenum Press (1971), 225-243
17. **Vavra, M. H.** The applicability of similarity parameters to the compressible flow in radial turbomachines Proc Ins Mech Eng Internal Thermodynamics (Turbomachine 1970), 118-132
18. **Watanabe, I. Ariga, I. and Mashimo, T.** Effect of dimensional parameters of impellers on performance characteristics of a radial-flow turbine Trans ASME J Eng Power (1971), 81-102
19. **Whitfield, A. and Baines, N. C.** Design of Radial Turbomachines Longman Scientific & Technical (1990)
20. **Whitfield, A.** Computer aided design and testing of radial and centrifugal impellers with straight and backward swept blades Proc Inst Mech Eng (1978), 89-100

# Appendix

### Matlab Code:

```
%Sample Matlab code for Major dimensions of Expansion Turbine and Blade
Profile%
%Author: Chandan Kumar Sahu
%NIT ROURKELA,INDIA%
clear;
%%
disp('Everything is in SI units');
%input Parameters%
Ns=0.54;                %specific speed%
Ds=3.4;                 %specific Diameter%
enth=38.7e+03;          %difference between enthalpy at inlet and isentropic
enthalpy at diffuser exit%
k2=1.03;                %kun-Sentz suggestion%
mtr=23.26e-03;          %mass flow rate%
pex=5.86;               %density at diffuser exit%
Ztr=10;                 %number of blades to be chosen%
ttr=0.0006;             %blade thickness%
g=1.4;                  %Specific Heat Ratio%
m=0.5;                  %polytropic index%
hex=90.11e+03;
Pex=1.5e+05;
k1=1;               %Initial Guess for k1%
%%
err=1;
%basic DimeNsioNs of Turbine Wheel%
while(abs(err)>0.01)
    Dh=k2*(enth);        %difference between enthalpy at inlet and isentropic
enthalpy at turbine exit%
    Qex=mtr/pex;         %discharge at diffuser exit%
    disp(['the value of k1 is:  ', num2str(k1)]);
    p3=pex/k1;           %density at turbine exit%
    Q3=k1*Qex;           %discharge at turbine exit%
    rps=Ns*((Dh)^(.75))*((Q3)^(-0.5));      %angular velocity in rad/sec%
    D2=Ds*(Q3^.5)*(Dh^(-0.25));              %inlet diameter in millimeter%
    Dtip=0.676*D2;                           %tip diameter at turbine exit%
    Dhub=.425*Dtip;                          %hub diameter at turbine exit%
    con1=(((16)*Q3)/((Dtip^2)-(Dhub^2)));
    con2=rps*pi*(Dtip+Dhub);
    con3=2*rps*Ztr*ttr;
    sub1=(con1)^2+(con2)^2-(con3)^2;
    sinb=(con2*con3+con1*sqrt(sub1))/((con1)^2+(con2)^2); %sine of beta mean%
    bm=asin(sinb);                 %the mean relative velocity angle%
    C3=0.25*rps*(Dtip+Dhub)*(tan(bm));     %exhaust velocity%
    U3=C3/(tan(bm));
    W3=sqrt(C3^2+U3^2);
    %%
    %Diffuser Design%
    DinD=D2+2*0.00025;      %Diameter of Diffuser Inlet%
    DthD=Dtip+2*0.0001;     %Diameter of the Throat of the Diffuser%
    minDexD=sqrt(4*Qex/(20*pi));
        if minDexD<D2
```

```
               DexD=input(['Enter a value for Diameter at diffuser exit more 
than ', num2str(D2), ' in meter : ']);
        else
              DexD=input(['Enter a value for Diameter at diffuser exit more 
than ', num2str(minDexD),' in meter: ']);
        end
    AexD=0.25*pi*DexD^2;            %Area of crosssection at diffuser exit%
    AthD=0.25*pi*DthD^2;            %Area of crosssection at throat%
    LdD=0.5*(DexD-DthD)/tand(5);%Length of the diverging section of diffuser%
    Cex=Qex/AexD;                   %discharge velocity at diffuser exit%
%thermodynamic state at wheel discharge%
    h0ex=hex+0.5*Cex^2;          %stagnation enthalpy at exit%
    P0ex=Pex+0.5*pex*Cex^2;
    h03=h0ex;
    s3=input(['calculate the entropy in J/Kg-K  corresponding to the 
stagnation enthalpy: ',...
        num2str(.001*h03), ' KJ/Kg and stagnation pressure: ', 
num2str(.00001*P0ex), 'bar :  ']);
    h3=h03-.5*C3^2;
    p3s=input(['calculate the density in Kg/m3 corresponding to the entropy: 
', ...
        num2str(.001*s3), ' KJ/Kg-K and static enthalpy: ', num2str(.001*h3), 
'KJ/Kg  :  ']);
    Cs3=input('Enter the corresponding velocity of sound in m/s:  ');
    err=p3-p3s;
    disp(['error in density is ', num2str(err)]);
    if(abs(err)>0.01)
        k1=k1+0.1*err;
    end
end
%%data input
%%variable initiation
%profile has been integrated from turbine exit to inlet-starting from zero at 
exit to inlet%
s3=0;               %meridional streamlength at turbine exit%
z3=0;               %blade height at turbine exit%
l3=0;               %characterstic angle-angle between meridional velocity 
and axial coordinate at turbine exit%
th3=0;              %tangential coordinate along meridional streamlength%
%%
%blade parameters
r2=D2/2               %inlet radius%
r3m=0.25*(Dhub+Dtip)        %mean exit radius=.5*(rtip+rhub)%
rm=r3m;            %mean radius initiated at exit%
b3=bm;             %mean relative velocity angle at exit%
b2=0.5*pi;          %mean relative velocity angle at inlet%
kh=5;               %free parameter%
ke=0.75;            %free parameter%
M3=W3/Cs3;          %Mach Number%
g=1.4;              %Specific Heat Ratio%
m=0.5;              %Polytropic Index%
%%
%iteration control
n=21;               %number of points%
r=r3m;              %blade radius initiation%
sr=1;             %initial guess for sratio%
err=(r2-r)/r2;    %control variable intiation%
```

```
%%
while(abs(err)>0.001)                %precision check%
    s2=sr*r3m;
    ds=s2/n;
    rcount=1;
    s=s3 ;
    l=l3;
    z=z3;
    r=r3m;
    th=th3 ;
    while(rcount<=n)%Coordinate calculation for 20 point on blade profile%
        %%
        % Mean Blade Profile Estimation %
            A=(((s/s2)*(kh+1)*csc(b2)+(csc(bm)-csc(b2))*(1-((1-
s/s2)^(kh+1))))/(kh*csc(b2)+csc(bm)))^ke;
            f1=sqrt((csc(bm))^2+A*((csc(b2))^2-(csc(bm))^2));
            f2=1/(csc(b2)+((csc(bm)-csc(b2))*((1-s/s2)^kh)));
            f3=f1*sqrt(1-(f2)^2);
            den1=r/(rm*tan(bm));
            Rm=(((f1*f2)/(den1-f3))^2)*(r/cos(l));
            dr=ds*sin(l);
            dz=ds*cos(l);
            dth=(sqrt(1-(f2)^2))/(r*f2)*ds;
            dl=ds/Rm;
            r=r+dr ;        %radial coordinate along meridional streamlength%
            z=z+dz  ;       %axial coordinate along meridional streamlength%
            th=th+dth;   %tangential coordinate along meridional treamlength%
            l=l+dl;
            radius(rcount)=r;
            height(rcount)=z;
            theta(rcount)=th;
            s=s+ds ;
            %%
             %Tip and hub profile estimation%
        Cm3=U3*tan(b3);
        W=Cm3*f1;
        U=U3*r3m/r;
        b=asin(f2);
        Cu=U-W*cos(b);
        Cm=Cm3*f1*f2;
        C=sqrt(Cm^2+Cu^2);
        frac=(U^2-W^2-U3^2+W3^2)/(Cm3^2);
        p=p3*(1+(M3^2)*((m-1)*g/(2*m))*frac)^((m-1)^(-1));
        wtr=(2*pi*r*sin(b)-Ztr*ttr)/Ztr;
        btr=mtr*sin(b)/(Ztr*wtr*p*Cm);

        rhub=(r-0.5*btr*cos(l));
        rtip=(r+0.5*btr*cos(l));
        XHUB(rcount)=1000*rhub*cos(th);
        YHUB(rcount)=1000*rhub*sin(th);
        XTIP(rcount)=1000*rtip*cos(th);
        YTIP(rcount)=1000*rtip*sin(th);
        rcount=rcount+1;
    end
     err=(r2-r)/r2;
     sr=sr*r2/r;
end
```

```
zrtheta=[height',radius',theta']
hubprofile=[XHUB', YHUB', 1000*height']
tipprofile=[XTIP', YTIP', 1000*height']
```

### *Result:*

The value of $k_1$ was found to be 1.1154 for an error limit of 0.001 on the density.

```
The value of k1 is:  1.1154
Enter a value for Diameter at diffuser exit more than 0.016011 in meter :
0.019
Read the recovery factor from the figure attached
calculate the entropy in J/Kg-K  corresponding to the stagnation enthalpy:
90.208 KJ/Kg and stagnation pressure: 1.5057bar :  5452.3
calculate the density in Kg/m3 corresponding to the entropy: 5.4523 KJ/Kg-
K and static enthalpy: 86.1634KJ/Kg  :  5.2448
Enter the corresponding velocity of sound in m/s:  184.4
error in density is 0.0088734
```

| hubprofile = | | | tipprofile = | | |
|---|---|---|---|---|---|
| 2.2760 | 0.3313 | 0.5679 | 5.3552 | 0.7796 | 0.5679 |
| 2.4175 | 0.6696 | 1.1358 | 5.0143 | 1.3888 | 1.1358 |
| 2.4646 | 0.9815 | 1.7037 | 4.7012 | 1.8722 | 1.7037 |
| 2.4617 | 1.2591 | 2.2716 | 4.4115 | 2.2564 | 2.2716 |
| 2.4317 | 1.5014 | 2.8394 | 4.1522 | 2.5638 | 2.8394 |
| 2.3916 | 1.7127 | 3.4071 | 3.9281 | 2.8131 | 3.4071 |
| 2.3542 | 1.8997 | 3.9742 | 3.7426 | 3.0200 | 3.9742 |
| 2.3295 | 2.0709 | 4.5400 | 3.5975 | 3.1981 | 4.5400 |
| 2.3252 | 2.2355 | 5.1035 | 3.4935 | 3.3587 | 5.1035 |
| 2.3480 | 2.4033 | 5.6628 | 3.4306 | 3.5114 | 5.6628 |
| 2.4035 | 2.5840 | 6.2151 | 3.4083 | 3.6642 | 6.2151 |
| 2.4967 | 2.7869 | 6.7566 | 3.4256 | 3.8238 | 6.7566 |
| 2.6315 | 3.0203 | 7.2822 | 3.4815 | 3.9958 | 7.2822 |
| 2.8104 | 3.2901 | 7.7856 | 3.5750 | 4.1853 | 7.7856 |
| 3.0337 | 3.5997 | 8.2595 | 3.7053 | 4.3966 | 8.2595 |
| 3.2991 | 3.9485 | 8.6961 | 3.8717 | 4.6338 | 8.6961 |
| 3.6017 | 4.3328 | 9.0878 | 4.0732 | 4.8999 | 9.0878 |
| 3.9343 | 4.7458 | 9.4288 | 4.3079 | 5.1965 | 9.4288 |
| 4.2886 | 5.1796 | 9.7154 | 4.5729 | 5.5229 | 9.7154 |
| 4.6564 | 5.6262 | 9.9474 | 4.8638 | 5.8768 | 9.9474 |
| 5.0308 | 6.0790 | 10.1277 | 5.1757 | 6.2541 | 10.1277 |